FINAL: ALGEBRA I

Date: **15th November 2017**
The Total points is **112** and the maximum you can score is **100** points.

A **ring** would mean a **commutative ring with identity**.

(1) (8+8+8+8=32 points) Mention all the options which are correct. No justification needed.
  (a) The following are isomorphic:
    (i) $(\mathbb{R}^*, \cdot)$, $\mathbb{C}^*, \cdot)$ as groups.
    (ii) $(\mathbb{R}, +)$, $(\mathbb{R}^{>0}, \cdot)$ as groups.
    (iii) $GL_n(\mathbb{F}_2)$ and $SL_n(\mathbb{F}_2)$ as groups.
  (b) The composition factors of a finite nontrivial solvable group are:
    (i) cyclic groups
    (ii) simple groups
    (iii) Alternating groups
    (iv) cyclic groups of prime orders
  (c) The number of groups of order 35 up to isomorphism are:
    (i) 0
    (ii) 1
    (iii) 2
    (iv) 3
  (d) Let G be a finite group, H be a subgroup and P a Sylow p-subgroup of G for a prime number p. Then
    (i) $H \cap P$ is a Sylow p-subgroup of H.
    (ii) $H \cap P$ is a Sylow p-subgroup of H if P is a normal subgroup.
    (iii) None of the above.

(2) (5+15=20 points) What is meant by a group G acts on a set S. Let n be a composite number and G be a group of order n such that G contains a subgroup of order k for every k which divides n. Show that G is not simple.

(3) (5+15=20 points) Define alternating group and simple group. Let p be an odd prime and $n \geq p$. Show that the alternating group A_n is generated by p-cycles.

(4) (5+15=20 points) Let R be a ring. Define flat R-module. Let A be a flat R-algebra and M be a flat A-module. Show that M is a flat R-module.

(5) (15+5=20 points) Let R be a subring of $\mathbb{C}[t]$ containing $\mathbb{C}$. Let $f(t) \in R$ be of degree $n \geq 1$. Show that $\mathbb{C}[t]$ as an R-module is generated by n elements. Is $\mathbb{C}[[t]]$ a finitely generated $\mathbb{C}[t]$-module?